INVESTIGATION OF THE HYDRODYNAMICS IN THE NEAR-WALL BOUNDARY LAYER OF A SEMI-INFINITE JET

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The velocity profiles and friction stresses at the wall in a semi-infinite jet are investigated.

The authors of a majority of the theoretical papers in this area utilize the method of joining solutions for the jet and near-wall boundary layers, and also assume the "one-seventh" velocity distribution law in the near-wall boundary layer as well as the Blasius relationship for τ_W. According to [1, 2] and ourselves (Fig. 1), the velocity profiles in the near-wall boundary layer differ considerably from the "one-seventh" law. The best agreement is obtained when the profile is approximated by the curve

$$u/u_m = (y/\delta)^{1/12}. \quad (1)$$

The difference between the velocity profile (1) and the curve constructed by the "one-seventh" law should be indicated as a change in τ_W [3]:

$$\tau_w = 0.00833\rho u_m^2 (u_m\delta/\nu)^{-2/13}. \quad (2)$$

Let us write the Karman integral relation as

$$\frac{d}{dx}\int_0^\delta \rho u^2 dy - u_m \frac{d}{dx}\int_0^\delta \rho u dy = -\tau_w. \quad (3)$$

Utilizing the dependences (1), (2) and integrating we reduce (3) to

$$\frac{6}{91}\bar{u}_m \frac{d\bar{\delta}}{d\bar{x}} - \frac{72}{91}\bar{\delta}\frac{d\bar{u}_m}{d\bar{x}} = 0{,}00833\bar{u}_m\left(\frac{u_m\delta}{\nu}\right)^{-2/13}. \quad (4)$$

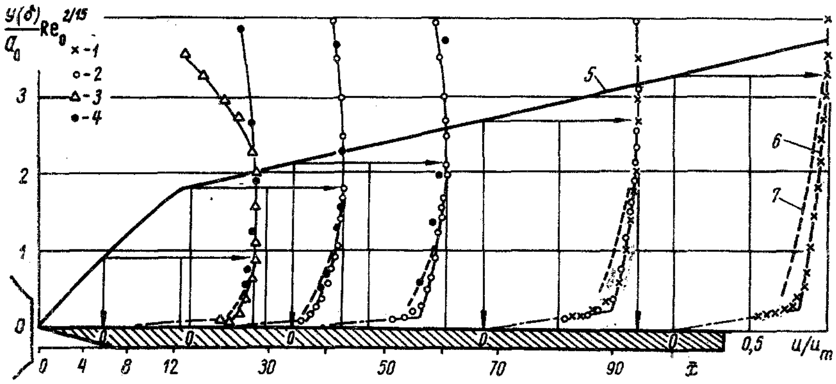

Fig. 1. Change in boundary layer thickness and velocity profiles along a wall: 1) a_0 = 4.5 mm, u_0 = 25 m/s; 2) 4.5 and 23; 3) 12 and 22, the author's tests; 4) 10 and 3.2, tests in [4]; 5) computations using (5) and (8); 6) velocity profile computed by means of (1); 7) the "one-seventh" law.

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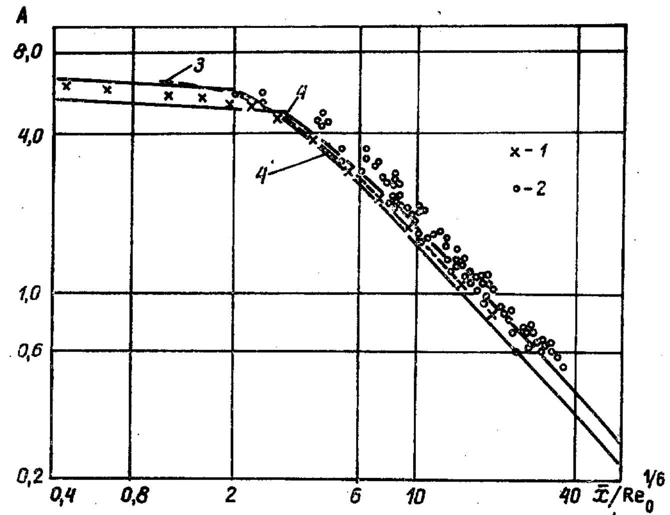

Fig. 2. Friction coefficient at the wall ($A = C_{f_0}\mathrm{Re}_0^{1/4}\cdot 10^2$): 1) author's tests; 2) tests of Meyers et al. [5]; 3) Sigalla's tests [6]; 4, 4') computations using (6) and (9) for $\mathrm{Re}_0 = 7000$ and 6000, respectively.

Initial Section. In the initial section of the jet $u_m \approx u_0$. Integrating (4) between x = 0 and x, we obtain

$$\bar{\delta} = 0.2\bar{x}^{13/15}\,\mathrm{Re}_0^{-2/15}, \tag{5}$$

and C_{f_0} can be determined from the equation

$$C_{f_0}/2 = \tau_w/\rho u_0^2 = 0.0107\bar{x}^{-2/15}\mathrm{Re}_0^{-2/15}. \tag{6}$$

Main Section. The change in u_m in the main section of a semi-infinite jet can be approximated by the formula ($\bar{x} \geq 12$)

$$\bar{u}_m = 3.6\bar{x}^{-1/2}, \tag{7}$$

which is in good agreement with all experiments in this region.

Integrating (4) taking account of (7) results in the expression

$$\bar{\delta} = (0.015\bar{x}^{14/15}\,\mathrm{Re}_0^{-2/13} + C)^{13/15}.$$

The constant of integration C is found from the condition that the boundary layer thicknesses in the initial and main sections of the jet are equal ($C = 1.73/\mathrm{Re}_0^{2/13}$) for $\bar{x} = 12$. Then

$$\bar{\delta} = 1.61\mathrm{Re}_0^{-2/15}\,(1 + 0.0088\bar{x}^{14/13})^{13/15}, \tag{8}$$

where C_{f_0} is determined from the equation

$$C_{f_0}/2 = 0.09\mathrm{Re}_0^{-2/15}\bar{x}^{-12/13}\,(1 + 0.0088\bar{x}^{14/13})^{-2/15}. \tag{9}$$

The change in the dimensionless velocity profiles for different distances from the nozzle is shown in Fig. 1. Superposed in this same figure is the change in near-wall boundary-layer thickness δ along the wall, as computed by means of (5) and (8). Besides our results, the results of the experiment in [4] conducted with a semi-infinite water jet are also shown in Fig. 1. A comparison between our experimental results and the tests in [4] and the theoretical dependences (5), (8), and (1) shows good agreement.

If an analogous computation of the boundary layer is performed by using the Blasius law for τ_w and $u/u_m = (y/\delta)^{1/7}$, then we have for the initial section of a semi-infinite jet:

$$\bar{\delta} = 0.37\,\mathrm{Re}_0^{-0.2}\bar{x}^{0.8} \tag{10}$$

and for the main section

$$\bar{\delta} = 2.28\,\mathrm{Re}_0^{-0.2}\,(1 + 0.014\bar{x}^{1.125})^{0.8}. \tag{11}$$

A comparison between (5), (8), (10), and (11) in the range of variation of the Reynolds criterion in which the experiments were conducted did not show any great discrepancies. However, the deviation of the test results from the "one-seventh" law is significant, and particularly at large distances $\bar{x}$ (Fig. 1).

Given in Fig. 2 is a comparison between computations utilizing (6) and (9) and our experimental results, the tests of Meyers et al. [5] and the Sigalla [6] experiments which we borrowed from [5]. As is seen from Fig. 2, the test results of Meyers et al. [5] lie somewhat above the theoretical curves.

NOTATION

a_0	is the width of nozzle slit;
x	is the running coordinate;
$\bar{x} = x/a_0$	is the dimensionless running coordinate;
u_0	is the escape velocity from nozzle orifice;
u_m	is the velocity on the outer boundary of the near-wall boundary layer;
$\bar{u}_m = u_m/u_0$	is the dimensionless outer boundary velocity;
$Re_0 = u_0 a_0/\nu$	is the Reynolds criterion constructed by means of the initial jet parameters.

LITERATURE CITED

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